PROBLEM SET 1

It's OK to co-operate with classmates on problem sets. If you get stuck on a problem, don't waste a lot of time on it --- you have better things to do.

The following problems from Starr's *General Equilibrium Theory*, *2nd edition*, are assigned.

2.9

3.5

4.7

4.8

5.3

5.2

5.5

In addition, two problems adapted from past quals are assigned, attached below.

This question is adapted from the June 2014 Micro Qual.

3. Consider an exchange economy with n households (consumers) and m goods. Assume that the households have identical preferences. Their preferences are represented by a single utility function $u : \mathbb{R}^m \to \mathbb{R}$ that is strictly increasing, homothetic, and strictly quasiconcave. The latter assumption means that, for consumption bundles x and x' with $u(x) = u(x')$, and for any $\alpha \in (0, 1)$,

$$u(\alpha x + (1 - \alpha)x') > u(x).$$

The households may or may not have different endowments. Let $e^i = (e^i_1, e^i_2, \ldots, e^i_m)$ be the endowment of household i, for $i = 1, 2, \ldots, n$. That is, household i's endowment of good k is e^i_k.

(a) Under what additional conditions (if any) does this economy have a perfectly competitive equilibrium? Is the equilibrium unique? Is the equilibrium efficient?

(b) Under what conditions does this economy have a competitive equilibrium in which no trade occurs?

(c) Suppose the government intervenes in the economy by taxing and redistribution goods before trade takes place. In particular, the government takes a fraction $\beta \in (0, 1)$ of every household's endowment of each good. The government then gives back to the households an equal share of the goods collected. Thus, after taxation and redistribution, household i has the following amount of good k:

$$(1 - \beta)e^i_k + \frac{1}{n}\sum_{j=1}^{n} \beta e^j_k.$$

Then the households trade in the marketplace. Is there a competitive equilibrium? If so, under what conditions is it efficient?

This question is taken from the September 2004 Micro Qual. You should be able to do 1, 2, ..., 5. Ignore 6.

Part 2

To assist you, restatements of the following concepts appear at the end of this part: competitive equilibrium, First Fundamental Theorem of Welfare Economics, Second Fundamental Theorem of Welfare Economics.

Consider a Robinson Crusoe economy with two goods x and y and a single input to production, L. Let L^x denote the amount of L used in producing x, L^y denote the amount of L used in producing y. The production relations are piecewise linear, represented as

$$x = F(L^x) = L^x, \text{ for } 55 \geq L^x \geq 0$$
$$x = F(L^x) = 55 + 2(L^x - 55) \text{ for } L^x > 55$$
$$y = F(L^y) = L^y, \text{ for } 55 \geq L^y \geq 0$$
$$y = F(L^y) = 55 + 2(L^y - 55) \text{ for } L^y > 55.$$

The resource constraint on the economy is $L^x + L^y = 100$ and $L^x,\ L^y \geq 0$.

Good x is priced at $p^x = 1$.

Good y is priced at $p^y = 1$.

L is priced at a wage rate $w = 1$.

The household budget constraint is $B = 100w + \pi = p^x x + p^y y$. In this expression, π denotes profit, which you can take to be zero for the allocation discussed below.

Case 1

Let the household utility function be $u(x, y) = xy$.

1. Show that the household's optimizing consumption plan subject to the budget constraint is (x, y) = (50, 50). At this allocation is the pricing at marginal cost? Is the allocation Pareto efficient?

2. Note that the allocation is not a competitive equilibrium. It is not profit maximizing for the given prices and production functions. Explain why.

3. Is this example a counterexample to the Second Fundamental Theorem of Welfare Economics ? Explain why or why not.

(Part II continues on the next page.)

Case 2

Now change the utility function to $v(x, y) = x + y$.

4. Is the allocation from Case 1 still optimizing for the household subject to the budget constraint? Is pricing at marginal cost?

5. Demonstrate that the allocation from Case 1 is not Pareto efficient. Is this situation a counterexample to the First Fundamental Theorem of Welfare Economics? Explain.

To prepare for question 6 we introduce two concepts.

Marginal cost pricing equilibrium: A market clearing allocation where all households optimize utility subject to budget constraint and all firms' prevailing output prices are marginal costs. Note that a competitive equilibrium is also a marginal cost pricing equilibrium.

Hotelling's Proposition: For any Pareto efficient allocation, there is a suitable choice of prices and endowments so that the allocation is a marginal cost pricing equilibrium.

6. Is Case 2 a counterexample to Hotelling's Proposition? Explain.

A few familiar definitions for your reference:

Competitive equilibrium: A price-guided, price-taking market clearing allocation where all households optimize utility subject to budget constraint and all firms optimize profit subject to technology and prevailing prices.

First Fundamental Theorem of Welfare Economics: Assume monotonicity of preferences. A competitive equilibrium allocation (if it exists) is Pareto efficient.

Second Fundamental Theorem of Welfare Economics: Assume the following properties of technology: convexity, closedness, possibility of inaction ($0 \in$ technology sets), irreversibility, no free lunch. Assume the following properties on households: possible consumption set is closed, bounded below, unbounded above, convex; preferences are monotone, convex, continuous. Then any Pareto efficient allocation can be supported as a competitive equilibrium subject to a redistribution of income (or redistribution of endowment and ownership shares of firms) and subject to a boundary condition on income.